SIDDHARTH GROUP OF INSTITUTIONS :: PUTTUR
Siddharth Nagar, Narayanavanam Road – 517583
QUESTION BANK (DESCRIPTIVE)

Subject with Code : Mathematics-II **(18HS0831)Course & Branch**: B.Tech – ALL

Year &Sem: I-II **Regulation:** R18

UNIT –I

1. a) Verify the Exactness of $(2x-y+1)\,dx+(2y-x-1)dy=0$ [2M]

   b) Solve $(x^2-ay)dx=(ax-y^2)dy.$ [2M]

   c) Solve$x\,dy-y\,dx=xy^2\,dx.$ [2M]

   d) Find the Integrating Factor (I.F) of $xlogx\frac{dy}{dx}+y=2logx.$ [2M]

   e) Solve $\frac{dy}{dx}+y=x$. [2M]

2. a) Solve $\frac{\mathrm{dy}}{\mathrm{dx}}+\frac{\mathrm{ycosx+siny+y}}{\mathrm{sinx+xcosy+x}}=0$. [5M]

   b) Solve $y(2x^2y+e^x)dx=(e^x+y^3)dy.$ [5M]

3. a)Solve $xydx-(x^2+2y^2)dy=0.$ [5M]

   b) Solve the D.E $y(2xy+e^x)dx-e^xdy=0$ [5M]

4. a) Solve $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2.$ [5M]

   b) Solve $\frac{\mathrm{dy}}{\mathrm{dx}}(\mathrm{x}^2\mathrm{y}^3+\mathrm{xy})=1$. [5M]

5. a) Solve $(1+y^2)dx=(tan^{-1}y-x)dy.$ [5M]

   b) Solve $\frac{dy}{dx}+y\tan x=y^2\sec x.$ [5M]

6. a) Solve Solve$x\frac{dy}{dx}+y=logx$. [5M]

   b) Solve $\mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}}+\mathrm{y}=\mathrm{x}^3\mathrm{y}^6$. [5M]

7. a) Solve $\frac{dy}{dx}+ycosx=y^3sin2x.$ [5M]

   b) Solve $\frac{dy}{dx}+2xy=e^{-x^2}.$ [5M]

8. a) Solve $p^2+2pycotx=y^2.$ [5M]

   b) Solve $y=psinp+cosp.$ [5M]

9. a) Solve $y=2px+p^n.$ [5M]

   b) Solve$x^2\left(\frac{dy}{dx}\right)^2+xy\frac{dy}{dx}-6y^2=0.$ [5M]

10. a) Solve $\mathrm{y}=2\mathrm{px}+\mathrm{y}^2\mathrm{p}^3.$ [5M]

    b) Solve $(px-y)(py+x)=a^2p.$

UNIT –II

1. a) Solve $(D^2+2D+1)y=0$ [2M]

b) P.T $\left[J_{\frac{1}{2}}(x)\right]^2+\left[J_{-\frac{1}{2}}(x)\right]^2=\frac{2}{\pi x}$ [2M]

c) Find Particular Integral of $(D^2+6D+9)y=2e^{-3x}$ [2M]

d) Write the formula for Bessel's function $J_n(x)$. [2M]

e) Find the Legendre's Polynomials $P_0(x)\ and\ P_1(x)$.. [2M]

2. a) Solve ($(D^2+4)y=e^x+\sin 2x$. [5M]

b) Solve $(D^2+1)y=\cos x$ by method of variation of parameters. [5M]

3. a) Solve $(D^2-5D+6)y=xe^{4x}$. [5M]

b) Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=(1+x)^2$ [5M]

4. a) Solve $(D^3+2D^2+D)y=x^3$. [5M]

b) Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$ [5M]

5. a) Solve $(D^2+a^2)y=\tan ax$ by method of variation of parameters. [5M]

b) Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ [5M]

6. a) Solve by method of variation of parameters $(D^2-2D)y=e^x\sin x$. [5M]

b) Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$. [5M]

7. Solve in series the equation $\frac{d^2y}{dx^2}+xy=0$ [10M]

8.a)Express the following in terms of Legendre's polynomials $f(x)=x^3+2x^2-x-3$. [5M]

b) Prove that $\frac{d}{dx}\left[x^nJ_n(x)\right]=x^nJ_{n-1}(x)$ [5M]

9.a) Using Rodrigue's formula P.T $\int_{-1}^{1}x^mP_n(x)\,dx=0\ if\quad m<n$ [5M]

b) Express $J_4(x)$ in terms of $J_0(x)\,\&\,J_1(x)$ [5M]

10. Prove that $J_{\frac{5}{2}}(x)=\frac{3}{x}\left[\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x}-\cos x\right)\right]-\sqrt{\frac{2}{\pi x}}\sin x$. [10M]

UNIT –III

1. a)Evaluate $\int_0^1 \int_0^x e^{x+y}\, dxdy.$ [2M]

   b) Evaluate $\int_0^1 \int_0^1 \frac{dx\, dy}{\sqrt{1-x^2}\sqrt{1-y^2}}$ [2M]

   c) Change the order of integration in $\int_0^1 \int_0^{2\sqrt{x}} f(x,y)dydx$. [2M]

   d) Evaluate $\int_0^{\pi} \int_0^{\sin\theta} rdrd\theta$ [2M]

   e) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dxdydz.$ [2M]

2. a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$ [5M]

b)Evaluate $\iint (x^2+y^2)dxdy$ in the positive quadrant for which $x+y \le 1.$ [5M]

3. a) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. [5M]

   b) Evaluate $\int_0^{\pi} \int_0^{a(1+\cos\theta)} rdrd\theta$ [5M]

4. a) Evaluate $\int_0^{a} \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$ [5M]

   b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dxdy$ by converting to polar coordinates. [5M]

5. a) Show that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$. [5M]

   b) Evaluate the integral by transforming into polar coordinates $\int_0^{a} \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\,dxdy$. [5M]

6. a) Calculate $\iint r^3 dr\, d\theta$ over the area included between the circles $r=2sin\theta \; and \; r=4sin\theta$.[5M]

   b) Evaluate the integral by changing the order of integration $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dydx.$ [5M]

7. Change the order of integration in $I=\int_0^1 \int_{x^2}^{2-x} (xy)dydx$ and hence evaluate the same. [10M]

8. a) By changing order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$. [5M]

   b) Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x+y+z)dxdydz$ [5M]

9. Evaluate $\int_0^{a} \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. [10M]

10. a) Calculate the volume of the solid bounded by the planes

$$x=0, y=0, x+y+z=a \; and \; z=0.$$ [5M]

   b) Evaluate the triple integral $\iiint xy^2zdxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$. [5M]

UNIT –IV
COMPLEX ANALYSIS- DIFFERENTATION

1. a) Write Cauchy's Riemann equations in Cartesian form. [10M]
   b) Write the formula for Harmonic function.
   c) Show that $f(z) = z^2$ is analytic function.
   d) Define Bilinear Transformation.
   e) Write Cauchy's Riemann equations in polar form.

2. a) Show that $u = \frac{1}{2} log(x^2 + y^2)$ is Harmonic. [5M]
   b) If $w = f(z)$ is analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|Real\, f(z)|^2 = 2|f^1(z)|^2$. [5M]
3. a) Find 'a' and 'b' if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic. Hence find $f(z)$ $interms\ of\ z.$ [5M]
   b) Find the analytic function whose imaginary part is $e^x(x\sin y + y\cos y)$. [5M]
4. a) Determine p such that the function $f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$. [5M]
   b) Find all the values of k, such that $f(x) = e^x(\cos ky + i\sin ky)$. [5M]
5. a) If $f(z) = u + iv$ is an analytic function of z and if $u + v = e^x(\cos y - \sin y)$, Find $f(z)$ in terms of z. [5M]
   b) Find an analytic function whose real part is $e^{-x}(x\sin y - y\cos y)$. [5M]
6. a) Show that $(z) = z + 2\bar{z}$ is not analytic anywhere in thecomplexplane. [5M]
   b) Show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$. [5M]
7. a) Find thebilinear transformationwhich maps the point's$(\infty, i, 0)$into thepoints$(0, i, \infty)$. [5M]
   b) Find the image of the triangular region with vertices at $(0,0)(1,0)(0,1)$ under the transformation $w = (1 - i)z + 3$. [5M]
8. a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. [5M]
   b) Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane, into a circle in the $w - plane$. [5M]
9. a) Find the bilinear transformation which maps the points $(\infty, i, 0)$ in to thepoints $(-1, -1, 1)$in w-plane. [5M]
   b) Find the bilinear transformation that maps the point's$(1, i, -1)$ in to the points $(2, i, -2)$in w-plane.[5M]
10. a) The image of infinite strip bounded by $x = 0\ \&\ x = \frac{\pi}{4}$ under thetransformation $w = \cos z$ [5M]
    b) Prove that the transformation $w = \sin z$maps the families of lines $x = y = constant$ into two families of confocalcentral conics. [5M]

UNIT –V

1. a) State Cauchy's theorem [2M]
   b) State Cauchy's integral formula [2M]
   c) State Cauchy's residue theorem [2M]
   d) Find the poles of the function $f(z)=\frac{z}{\cos z}$ [2M]
   e) Find the residue of $f(z)=\frac{e^z}{z^5}$ [2M]

2. a) Evaluate the line integral $\int_c \left(y-x-3x^2 i\right)dz$ where c consists of the line segments from $z=0$ to $z=i$ and the other from $z=i$ to $z=i+1$. [5M]
   b) Evaluate $\int_0^{1+3i}\left(x^2-iy\right)dz$ along the path $y=x$. [5M]

3. Verify Cauchy's theorem for the function $f(z)=3z^2+iz-4$ if c is the square with vertices at $1\pm i \; and \; -1\pm i$. [10M]

4. a) Evaluate using Cauchy's integral formula $\int_c \frac{\sin^6 z}{\left(z-\frac{\pi}{2}\right)^3}dz$ around the circle $c:|z|=1$. [5M]
   b) Evaluate $\int_c \frac{\log dz}{(z-1)^3}$ where $c:|z-1|=\frac{1}{2}$ using Cauchy's integral formula [5M]

5. a) Expand $f(z)=\sin z$ in Taylor's series about $z=\frac{\pi}{4}$ [5M]
   b) Expand $f(z)=\log z$ in Taylor's series about $z=1$ [5M]

6. a) Find the Laurent series expansion of the function $f(z)=\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3<|z+2|<5$. [5M]
   b) Find the Laurent series of the function $f(z)=\frac{z}{(z+1)(z+2)}$ about $z=-2$. [5M]

7. a) Determine the poles of the function $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and the residues ateachpole [5M]
   b) Find the residue of the function $f(z)=\frac{1}{(z^2+4)^2}$ where c is $|z-i|=2$ [5M]

8. a) Evaluate $\int_c \frac{dz}{z^3(z+4)}$ $where\ c\ is\ |z|=2.$ [5M]
   b) Determine the poles and residues of $tanhz$. [5M]

9. Evaluate $\int_0^{2\pi}\frac{1}{a+b\cos\theta}d\theta=,\frac{\pi}{\sqrt{a^2-b^2}},a>b>0$ [10M]

10. Show that $\int_0^{2\pi}\frac{d\theta}{1+a^2-2acos\theta}=\frac{2\pi}{1-a^2}$, $0<a<1.$ [10M]